Indian Statistical Institute
M.Math I Year
First Semester Examination, 2005-2006
General Topology

Time: 3 hrs Date:25-11-05

Attempt any five questions. All questions carry equal marks. Any result proved in the class may be cited and used without proof.

1. a) Let X be compact and Hausdorff, $A \subsetneq X$ be closed. Show that X/A is homeomorphic to the one-point compactification of $X - A$.

   b) Describe explicitly the quotient topology on the quotient group $\mathbb{R}/\mathbb{Q}$, $\mathbb{R}$ being the real line, $\mathbb{Q}$ the set of rationals, treated as a subgroup of the group $(\mathbb{R}, +)$.

2. a) Prove that $GL(n, \mathbb{C})$ is path connected (hint; use the polynomial $p(z) = \det((1-z)I + zA)$ for $A \in GL(n, \mathbb{C})$).

   b) Prove that any discrete subgroup of $\mathcal{S}^1$ must necessarily be finite cyclic.

3. a) Let X be any space. Show that CX, the cone over X is contractible.

   b) Show that $\mathcal{S}^{n-1}$ is a deformation retract of $\mathcal{S}^n - \{N, \mathcal{S}\}, N$ and $\mathcal{S}$ being the north and south poles of $\mathcal{S}^n$ respectively.

4. Let $f, g : X \to \mathcal{S}^n$ be continuous maps with $f(x) \neq -g(x)\ \forall\ x \in X$. Prove that $f \simeq g$.

5. Let X be a space. Then show that X is path connected if and only if all constant maps: $X \to X$ are homotopic to each other.

6. Let $R_\theta : \mathcal{S}^1 \to \mathcal{S}^1$ be a rotation by angle θ. Show that R_θ is homotopic to the identity map: $\mathcal{S}^1 \to \mathcal{S}^1$.

7. Let G be a connected group, H a discrete normal subgroup. Prove that $H \subseteq Z(G)$, the centre of G.